8.75
23

November 16, 2000 | MATHEMATICS 110 (31) Quiz #3 | Total Marks - ~~25~~ Time: 45 minutes

Last Name: ______ Student Number: ______

[8] 1. Find all the critical numbers of the following functions. Show all your work.

a) $f(x) = \sqrt{x}(1-x)$

$D = (-\infty, 1) \cap (1, \infty)$

$f'(x) = \frac{dx^{1/2}}{dx}(1-x) + \frac{d(1-x)}{dx} x^{1/2}$

$= \frac{1}{2}x^{-1/2}(1-x) + (0)$

$= \frac{1}{2}x^{-1/2}(1-x) = 0$

$x = 1$

$x^{1/2} = 2$

$\sqrt{x} = 2$

$\frac{1}{\sqrt{x}} = 4$

$\frac{1}{2}x^{-1/2} = 0$

$\frac{1}{\sqrt{x}} = \frac{2}{1}$

Crit # is $x = \frac{1}{4}$

-1.5

b) $f(x) = \dfrac{x}{1+x}$

$D = (-\infty, -1) \cap (-1, \infty)$

Crit # $x = 1$ and 0

$f'(x) = \dfrac{\frac{dx}{dx}(1+x) - x\frac{d(1+x)}{dx}}{(1+x)^2}$

$-x = 0$

$1 + x = 0$

$x = 1$

-1.5

$\dfrac{(1+x) - x}{(1+x)^2} = \cancel{\dfrac{-x}{1+x}}$

c) $f(x) = 5 + x \ln x$

Crit # $x = 0$ and

$f'(x) = \ln x + x$

$\ln x = 0 \to 0$

-2

d) $f(x) = \cos x + \sin x, \quad x \in [-\pi, \pi]$

$D [-\pi, \pi]$

$f'(x) = -\sin x + \cos x$

+1.0

$\cos x = 0$

-1

$-\sin x = 0$

Crit # $x = 0$ and $-\pi$

[3] 2. For each of the following, if possible, sketch a graph of a function which has the stated property (if it is not possible to do so, explain why):

a) A continuous function which has no absolute extreme values.

b) A discontinuous function which has both an absolute maximum and an absolute minimum.

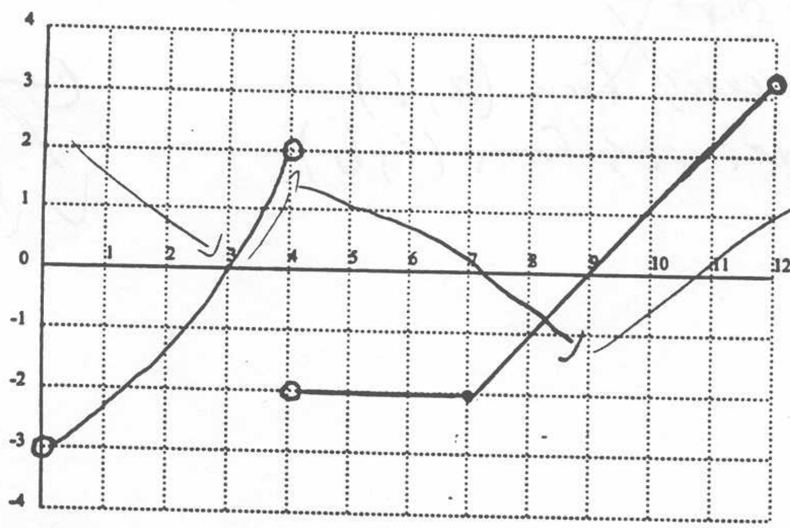
c) A continuous function which has no absolute extreme values on $[-1, 1]$.

-3

y (b) y (c) y

[3] 3. Consider the following **GRAPH OF THE DERIVATIVE** f' of a function f whose domain is (0,12):

Graph of f'



Answer the following questions in the space provided. Mark your answers clearly.

a) What are the critical numbers, if any, of the function f?

$x = 3$ and 9 and 4

b) On which interval(s) is f increasing and on which interval(s) is f decreasing?

increasing [3,4] ∩ [9,12)

−0.25

decreasing (0,3] ∩ [4,9]

c) At what values of x, if any, does f have local maxima or minima?

local Max : $x=4$

local Min = $x=3$ and $x=9$

[1] 4. State the Mean Value Theorem.

(1) Cont$^{\pm}$ [a,b]

−0.5 (2) diff$^{\pm}$ (a,b)

(3) Slope of f is equal to the Slope of f' Somewhere in [a,b]

[3] 5. Find the absolute extreme values for $f(x) = \dfrac{x}{x^2+4}$ on the interval $[0,2]$.

$f'(x) = 1(x^2+4) - (x)(2x)$ $D = \mathbb{R} \neq \pm 2$

[2] 6. **Prove that if f has a local maximum at c, and if $f'(c)$ exists, then $f'(c) = 0$, (i.e. prove part of Fermat's Theorem).**

① $[a,b]$ is cont^s

② (a,b) is diff^{ble}

③ it must increas from (a,c)

it must decreas from (c,b)

-2

[5] 7. **Consider the function**

$$f(x) = xe^{2x}$$

Answer the following questions in the space provided. Mark your answers clearly.

a) **What is the domain of f?**

b) **Find any x or y-intercepts for $y = f(x)$.**

c) **Determine any intervals where f is increasing or where f is decreasing.**

d) **Does f have any local extreme values? Justify your answer.**

1 $D = (-\infty, \infty)$

1 let $x = 0$

0.5 $y = 0e^{2x}$

$= 0$

y intercept at 0

let $y = 0$

$0 = xe^{2x}$

$\frac{0}{x} = e^{2x}$

ⓒ $f'(x) = e^{2x} + (x)(e^{2x})(2)$

$= (e^{4x})(x)(2)x$

e^{4x} — + + + + + + + +

x - - - - - - + + + +

f'

$-4 \quad 0 \quad 2 \quad 4$

-1

it is increasing $(0, \infty)$

it is decreasing $(-\infty, 0)$

ⓓ it has a min at $x = 0$.

this is because it is decreasing

21.5

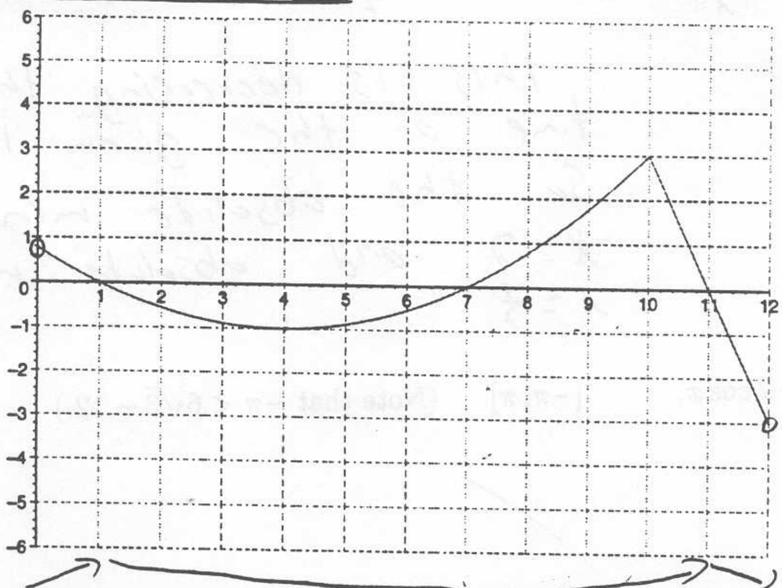
November 30, 2000 MATHEMATICS 110 (31) Total Marks - 35

Midterm #3 Time: 75 minutes

Last Name: ___________________ Student Number: __________

10 [10] 1. Consider the following **GRAPH OF THE DERIVATIVE** f' of a function f whose **DOMAIN** is **(0,12)**:

Graph of f'



Answer the following questions **about** f (not f') in the space provided.

a) What are the critical numbers, if any, of the function f?

$f'=0$ 1, 7, 11 are Crit #'s

b) On which interval(s) is f increasing and on which interval(s) is f decreasing?

increasing $f'>0 \Rightarrow (0,1] \cup [7,11]$

Decreasing $f'<0 \Rightarrow [1,7] \cup [11,12)$

c) At what values of x, if any, does f have local maxima or minima?

has a local max at $x=1$ and $x=11$

has a local min at $x=7$

d) On which interval(s) is f concave down and on which interval(s) is f concave up?

Concave down $(0,4] \cup [10,12)$ $f''<0$

concave up $[4,10]$ $f''>0$

e) At what values of x, if any, does f have a point of inflection?

$x=4$ and $x=10$

f) Given that $f(1)=f(10)=0$, $f(7)=-4$, $f(4)=-2$, $f(11)=1.5$, sketch

4 a graph of $y=f(x)$.

| [4]

2. Find the absolute maximum and minimum values of the following functions on the given intervals:

a) $f(x) = x^2 + \frac{2}{x}$, $[1/2, 2]$

$2x^3$

$\frac{-2}{x^2}$

$f'(x) = 2x + \frac{-2}{x^2}$

$f(x)$ $\frac{1}{2}$ 2

-1.5 $0 = 2x - \frac{2}{x^2}$

$2x - 2x^{-2} = 0$

$2(x - x^{-2}) = 0$

$x - x^{-2} = -2$

$x = -2$ $-x^{-2} = 2$ $x = \frac{1}{2}$

$x^{-2} = 2$

this is decreasing the whole time on the given intervals So the absolute min is $x = 2$ and absolute max is $x = \frac{1}{2}$.

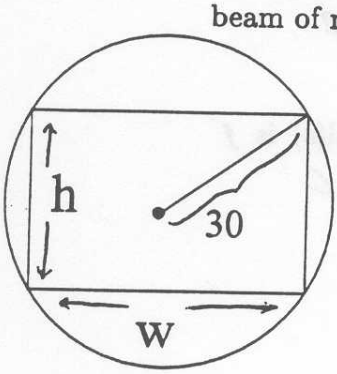
b) $f(x) = x - 2\cos x$, $[-\pi, \pi]$ (Note that $-\pi < 6\sqrt{3} - 12$.)

$f'(x) = 1 - 2(-\sin x)$

-1.5

0 [4]

3. Suppose a rectangular beam is cut from a cylindrical log of radius 30cm. The strength of the beam of width w and height h is given by kwh^2 for some fixed constant $k > 0$. (See the Figure below.) Find the width and height of the beam of maximum strength.



refer to solution set

$\frac{U^2}{dU} \cdot \frac{dU}{dx}$ $U = 1-x^2$

$\frac{dU}{dx} = -2x$

$2(1-x^2)(-2x)$

[12] 4. Find the following for

$$f(x) = \frac{2}{1-x^2}. \quad \frac{f}{g} \quad f'g - g'f$$

a) Domain of f.

b) All vertical and horizontal asymptotes for f and the associated limits.

c) All critical numbers for f.

d) The interval(s) where f is increasing and the interval(s) where f is decreasing.

e) The locations of any local maxima or minima.

f) The interval(s) where f is concave up and the interval(s) where f is concave down.

g) Any points of inflection for f.

h) Use this information to sketch a graph of f (there is space on the next page).

$-2(4)+16$
$-8+16$
$-2+16$

$\frac{2}{1-1^2} = \frac{2}{0}$ $x \neq 1$ $D\ (-\infty, 1)(1, \infty)$ $-1/2$

$\lim_{x\to 1^+} \frac{2}{1-x^2} = -\infty$ ✓ $\lim_{x\to 1^-} \frac{2}{1-x^2} = \infty$ ✓ a virtical asymtote at $x = 1$ and $x = -1$ ✓ $-1/2$

$\lim_{x\to -1^+} \frac{2}{1-x^2} = -\infty$ ✗ $\lim_{x\to -1^-} \frac{2}{1-x^2} = \infty$ ✗

$\lim_{x\to\infty} \frac{\frac{2}{x^2}}{\frac{1}{x^2}-\frac{x^2}{x^2}} = 0$ ✓ $\lim_{x\to-\infty} \frac{\frac{2}{x^2}}{\frac{1}{x^2}-\frac{x^2}{x^2}} = 0$ ✓ a horizontal asymtote at $x = 0$. $-1/4$

must be in D $f'(x) = \frac{0-(2)(-2x)}{(1-x^2)^2} = \frac{-(-2)(2x)}{(1-x^2)^2} = \frac{-2(2x)}{1-2x^2+x^4}$ $-1/2$

$0 = \frac{-2(2x)}{1-2x^2-x^4}$ $1-2x^2-x^4 = -2(2x)$ $\frac{1-2x^2-x^4}{2x} = -2$

$x = 0$ is Crit # ✓

-2	$-$	$-$	$-$	$-$	$-$
$2x$	$-$	$-$	0 $+$	$+$	$+$
$1-2x^2+x^4$	$+$	$-$	$+$	$-$	$+$
$f(x)$	↗	↘	↗	↘	
	-1		0	1	

f is increasing on $(-\infty, -1] \cup [0, 1)$.
f is decreasing on $[-1, 0] \cup (1, \infty)$ -1

) their is a local minima at $x = 0$ ✓ because it goes from decreasing to increasing at that point

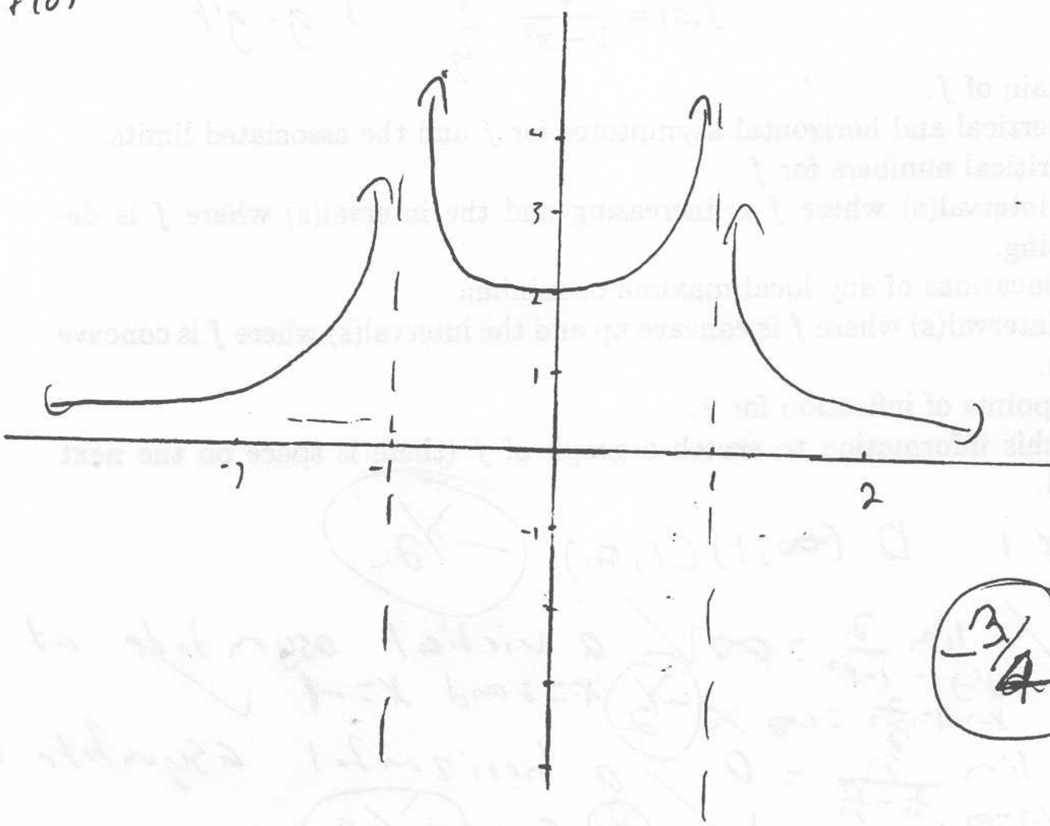
) $((0)(2x) + (-2)(2))(1-x^2)^2 = -2x(1-x^2)(-2x)$ $(-2)(2x)$

g) No Points of reflection

H) $\frac{2}{1-0} = f(0)$

$\frac{2}{1} = f(0)$

$= f(0)$



$\frac{3}{4}$

[5] 5. The following question deals with Newton's Method.

a) State the general formula for obtaining x_{n+1} from x_n using Newton's Method for finding a root of $f(x) = 0$.

b) Obtain an estimate for a solution to $f(x) = \cos x - \sin x = 0$ in $[0, \pi/2]$ using two steps (i.e. find x_3) of Newton's Method with $x_1 = 0$. Note that $\dfrac{1-\tan(1)}{1+\tan(1)} \approx -.21795$.

c) Explain how the approach used in d) could be used to get an estimate for π.

$0,1$ $1,0$ $(\cos, \sin 1$

a) $x_1 - \frac{f(x_1)}{f'(x_1)} = x_2 \Rightarrow x_n - \frac{f(x_n)}{f'(x_n)} = x_{n+1}$ $f'(x_n) \neq 0$ $f'(x) = -\sin x - \cos x$

b) $= 0 - \frac{\cos 0 - \sin 0}{-\sin 0 - \cos 0} = -\frac{1-0}{0-1} = \frac{-1}{-1} = 1 = x_2$

$1 - \frac{\cos 1 - \sin 1}{\sin 1 - \cos 1}$ $\times$ $1 - (\frac{1}{\tan 1} - \tan 1)$ $= \frac{1 - \tan 1}{1 + \tan 1} \approx -.21795 = x_3$

$x_1 = 0 \quad x_2 = 1 \quad x_3 = -.21795$

$-\frac{1}{2}$